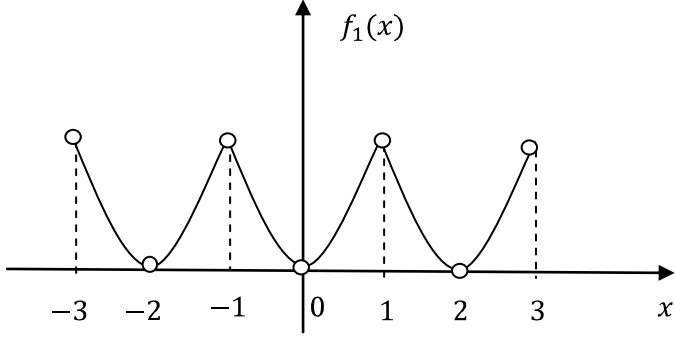
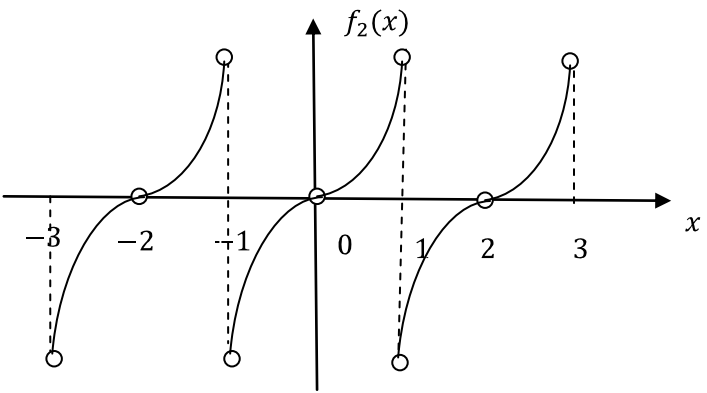
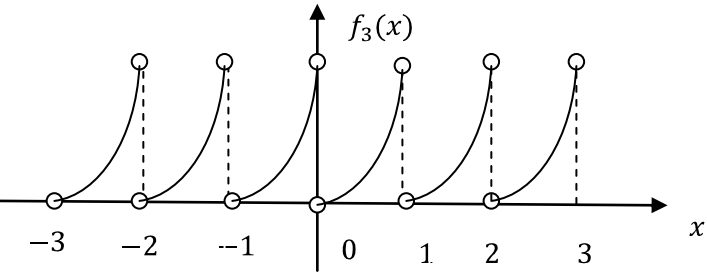
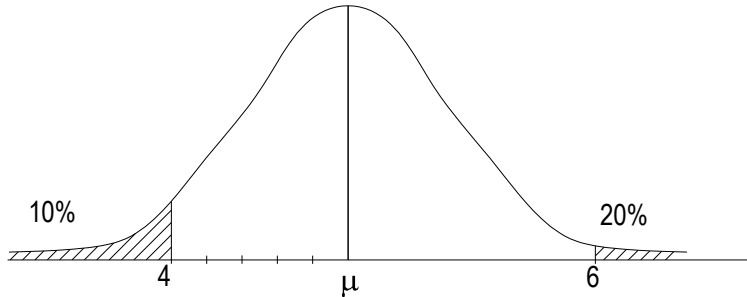


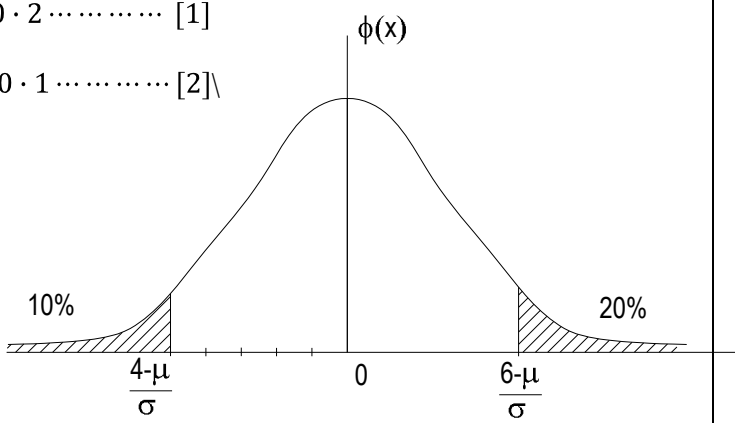
QNo	Answer	Marks	Comments
1	<p>(a) $f(x) = x^2 \quad x \in (0, 1)$</p> <p>(i) Define $f_1(x) = \begin{cases} x^2 & , x \in (0, 1) \\ -x^2 & , x \in (-1, 0) \end{cases}$ and $f_1(x) = f_1(x + 2k)$ where $k \in \mathbb{Z}$</p> <p>(ii) Define $f_2(x) = \begin{cases} x^2 & , x \in (0, 1) \\ x^2 & , x \in (-1, 0) \end{cases}$ and $f_2(x) = f_2(x + 2k)$ where $k \in \mathbb{Z}$</p> <p>(iii) Define $f(x) = x^2 \quad x \in (0, 1)$ and $f_3(x) = f_3(x + k)$ where $k \in \mathbb{Z}$</p>	40	
(b)	<p>(i)</p> 	10	
(ii)		10	
(iii)		10	

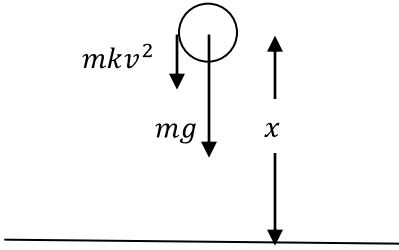
QNo	Answer	Marks	Comments
1	<p>(c) (i) Since f_1 is odd function a_0 and a_n are zero.</p> $b_n = 2 \int_0^1 x^2 \sin n\pi x dx$ $= 2 \left[-\frac{1}{n\pi} x^2 \cos n\pi x + \frac{1}{n^2 \pi^2} 2x \sin n\pi x + \frac{1}{n^3 \pi^3} \cos n\pi x \right]_0^1$ $= 2 \left[-\frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n^3 \pi^3} - \frac{1}{n^3 \pi^3} \right] \text{ Where } n = 1, 2, 3, \dots$ $f(x) = \sum_{n=1}^{\infty} 2 \left[-\frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n^3 \pi^3} - \frac{1}{n^3 \pi^3} \right] \sin n\pi x$ $= \sum_{n=1}^{\infty} 2 \left[-\frac{1}{n^3 \pi^3} - \frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n^3 \pi^3} \right] \sin n\pi x$	40	
	<p>(ii) Since f_2 is even function b_n is zero. $a_0 = 2 \int_0^1 x^2 dx = 2 \left[\frac{x^3}{3} \right]_0^1 = \frac{2}{3}$</p> $a_n = 2 \int_0^1 x^2 \cos n\pi x dx$ $= 2 \left[\frac{1}{n\pi} x^2 \sin n\pi x + \frac{1}{n^2 \pi^2} 2x \cos n\pi x - \frac{1}{n^3 \pi^3} \sin n\pi x \right]_0^1 = \frac{(-1)^n}{n^2 \pi^2}$ $f(x) = \frac{1}{3} + \sum_{r=1}^{\infty} 2 \frac{(-1)^n}{n^2 \pi^2} \cos n\pi x$	40	
	<p>(iii) $a_0 = 2 \int_0^1 x^2 dx = 2 \left[\frac{x^3}{3} \right]_0^1 = \frac{2}{3}$</p> $a_n = 2 \int_0^1 x^2 \cos 2n\pi x dx$ $= 2 \left[\frac{1}{2n\pi} x^2 \sin 2n\pi x + \frac{1}{4n^2 \pi^2} 2x \cos 2n\pi x - \frac{1}{8n^3 \pi^3} \sin 2n\pi x \right]_0^1 = \frac{1}{n^2 \pi^2}$ $b_n = 2 \int_0^1 x^2 \sin 2n\pi x dx$ $= 2 \left[-\frac{1}{2n\pi} x^2 \cos 2n\pi x + \frac{1}{4n^2 \pi^2} 2x \sin 2n\pi x + \frac{1}{8n^3 \pi^3} \cos 2n\pi x \right]_0^1$ $= -\frac{1}{n\pi} \text{ Where } n = 1, 2, 3, \dots$ $f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \left(\frac{1}{n^2 \pi^2} \cos 2n\pi x - \frac{1}{n\pi} \sin 2n\pi x \right)$	50	

QNo	Answer	Marks	Comments
2	(a) $f(x) = \frac{1}{x} = x^{-1} \quad f^1(x) = (-1)x^{-2}$ $f^2(x) = (-1)(-2)x^{-3}$ $f^3(x) = (-1)(-2)(-3)x^{-4}$ $f^n(x) = (-1)(-2)(-3) \dots (-n)x^{-n} \quad \forall n \in \mathbb{Z}_0^+$ $f^n(x) = \frac{n!(-1)^n}{x^{n+1}}$ $f^n(3) = \frac{n!(-1)^n}{3^{n+1}}$ $f(x) = \sum_{n=0}^{\infty} \frac{f^n(3)(x-3)^n}{n!}$ $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n! (x-3)^n}{3^{n+1} n!}$ $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{3^{n+1}}$	35	
	<p>The above series is a geometric series with the common ratio $\frac{3-x}{3}$ and the first term $\frac{1}{3}$.</p> <p>The series converges if and only if $\left \frac{3-x}{3} \right < 1$</p> $0 < x < 3$ <p>Therefore the convergence is $0 < x < 3$.</p> <p>The series converges to $\frac{1}{3} \frac{1}{1 - \frac{(3-x)}{3}} = \frac{1}{x}$</p>	35	
	(b) $\frac{d(v \times (r \times v))}{dt} = \frac{d((v \cdot v)r - (v \cdot r)v)}{dt}$ $= \left(v \cdot \frac{dv}{dt} + v \cdot \frac{dv}{dt} \right) r + (v \cdot v) \frac{dr}{dt} - \left(v \cdot \frac{dr}{dt} + \frac{dv}{dt} \cdot r \right) v - (v \cdot r) \frac{dv}{dt}$ $= 2(v \cdot a)r + v^2 v - (v^2 + a \cdot r)v - (v \cdot r)v$ $= 2(v \cdot a)r - (a \cdot r)v - (v \cdot r)v$	50	

QNo	Answer	Marks	Comments
2	(c) $\mathbf{f}(t) = \frac{e^t \sin t}{\sqrt{1+e^{2t}}} \mathbf{i} + \frac{e^t \cos t}{\sqrt{1+e^{2t}}} \mathbf{j} + \frac{1}{\sqrt{1+e^{2t}}} \mathbf{k}$ $ \mathbf{f}(t) = \sqrt{\left(\frac{e^t \sin t}{\sqrt{1+e^{2t}}}\right)^2 + \left(\frac{e^t \cos t}{\sqrt{1+e^{2t}}}\right)^2 + \left(\frac{1}{\sqrt{1+e^{2t}}}\right)^2}$ $ \mathbf{f}(t) = \sqrt{\frac{e^{2t} \sin^2 t + e^{2t} \cos^2 t + 1}{1+e^{2t}}}$ $ \mathbf{f}(t) = 1$ $ \mathbf{f}(t) ^2 = 1$ $\mathbf{f}(t) \cdot \mathbf{f}(t) = 1$ $\frac{d\mathbf{f}(t)}{dt} \cdot \mathbf{f}(t) + \mathbf{f}(t) \cdot \frac{d\mathbf{f}(t)}{dt} = 0$ $2\mathbf{f}(t) \cdot \frac{d\mathbf{f}(t)}{dt} = 0$ $\mathbf{f}(t) \cdot \frac{d\mathbf{f}(t)}{dt} = 0$ <p>$\mathbf{f}(t)$ is perpendicular to $\frac{d\mathbf{f}(t)}{dt}$</p> $\mathbf{f}(t) \cdot \frac{d^2 \mathbf{f}(t)}{dt^2} + \frac{d\mathbf{f}(t)}{dt} \cdot \frac{d\mathbf{f}(t)}{dt} = 0$ $\mathbf{f}(t) \cdot \frac{d^2 \mathbf{f}(t)}{dt^2} + \left \frac{d\mathbf{f}(t)}{dt} \right ^2 = 0$ $\mathbf{f}(t) \cdot \frac{d^2 \mathbf{f}(t)}{dt^2} = - \left \frac{d\mathbf{f}(t)}{dt} \right ^2$	80	

QNo	Answer	Marks	Comments
3	(a) Suppose the random variable X is the time taken to germinate a one seed  <p>Then $P(X > 6) = 0.2$ and $P(X < 4) = 0.1$</p> <p>Taking μ and σ^2 as mean and variance of X $Z = \frac{X-\mu}{\sigma}$</p>	140	

QNo	Answer	Marks	Comments
	<p>Using the standard normal variable Z</p> $P\left(Z > \frac{6-\mu}{\sigma}\right) = 0.2 \dots\dots\dots [1]$ $P\left(Z < \frac{4-\mu}{\sigma}\right) = 0.1 \dots\dots\dots [2]$  <p>Using [1] $P\left(Z < \frac{6-\mu}{\sigma}\right) = 0.8 \dots\dots\dots [3]$</p> <p>Since $\frac{4-\mu}{\sigma}$ is less than the mean value, so it is negative</p> <p>From the symmetry of the z - curve</p> $P\left(Z < \frac{4-\mu}{\sigma}\right) = P\left(Z > \frac{\mu-4}{\sigma}\right) = 0.1$ $P\left(Z < \frac{\mu-4}{\sigma}\right) = 0.9 \dots\dots\dots [4]$ <p>From the z - tables taking the nearest value of z giving these probabilities</p> <p>From equation [3] $\frac{6-\mu}{\sigma} = 0.84 \dots\dots\dots [5]$</p> <p>From equation [4] $\frac{\mu-4}{\sigma} = 1.28 \dots\dots\dots [6]$</p> <p>From the equations [5] and [6] $\mu = 5.208$ and $\sigma = 0.943$</p>		
(b)	<p>$\sigma = 6 \text{ cm}$ and $\mu = 140 \text{ cm}$</p> <p>Let X be the height of a tree then</p> $P(x < 145) = P\left(\frac{X - 140}{6} < \frac{145 - 140}{6}\right) = P\left(Z < \frac{5}{6}\right)$ $P(x < 145) = P\left(Z < \frac{5}{6}\right) = P(Z < 0.833) = 0.7976$ <p>Let Y be the number of trees in the sample that is less than 145 cm tall. Then Y can the value in the set $\{0, 1, 2, 3, 4, 5\}$</p> <p>Clearly Y has a Binomial distribution with $n = 5$ and $p = 0.7976$</p> $P(y = 5) = (0.7976)^5 = 0.323$ $P(y = 3) = {}^5C_3(1 - 0.7976)^2(0.7976)^3 = 0.2079$	60	

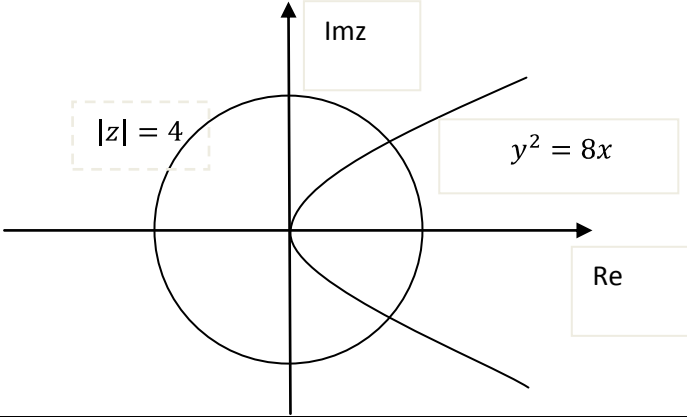
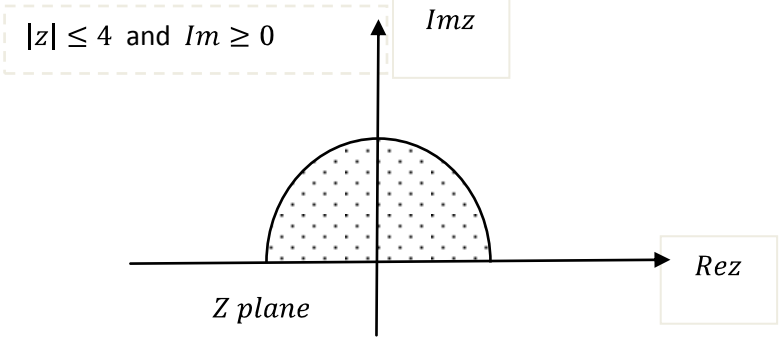
QNo	Answer	Marks	Comments
4	<p>(a)</p>  <p>Applying $\uparrow F = ma$</p> $-mg - mkv^2 = mv \frac{dv}{dx}$ $dx = -\frac{v}{g + kv^2} dv$ $\int_0^h dx = -\int_u^0 \frac{v}{g + kv^2} dv = \int_0^u \frac{v}{g + kv^2} dv$ $[x]_0^h = \left[\frac{1}{2k} \ln(g + kv^2) \right]_0^u$ $h = \frac{1}{2k} \ln(g + ku^2) - \frac{1}{2k} \ln g$ $= \frac{1}{2k} \ln \left(1 + \frac{ku^2}{g} \right)$	60	
	<p>(b) Applying mechanical energy conservation law</p> $\frac{1}{2} \frac{4}{3} ma^2 \dot{\theta}^2 - mg a \cos \theta = \frac{1}{2} \frac{4}{3} ma^2 \omega^2 - mg a$ $\dot{\theta}^2 = \omega^2 - \frac{3g}{2a} (1 - \cos \theta) \dots \dots [1]$	35	
	$\dot{\theta}^2 = \omega^2 - \frac{3g}{2a} + \frac{3g}{2a} \cos \theta \text{ where } 0 \leq \theta \leq 2\pi$ $\dot{\theta}^2_{min} = \omega^2 - \frac{3g}{2a} + \frac{3g}{2a} (-1) \quad \dot{\theta}^2_{min} = \omega^2 - \frac{3g}{a}$ <p>For the complete revolutions</p> $\dot{\theta}^2_{min} \geq 0 \quad \omega^2 \geq \frac{3g}{a}$ $\omega \geq \sqrt{\frac{3g}{a}} \text{ the minimum value for complete revolutions } \omega_0 = \sqrt{\frac{3g}{a}}$	35	
	<p>From equation [1] $\dot{\theta}^2 = \omega_0^2 - \frac{3g}{2a} 2 \sin^2 \left(\frac{\theta}{2} \right)$</p> $\dot{\theta}^2 = \frac{3g}{a} - \frac{3g}{a} \sin^2 \left(\frac{\theta}{2} \right)$	70	

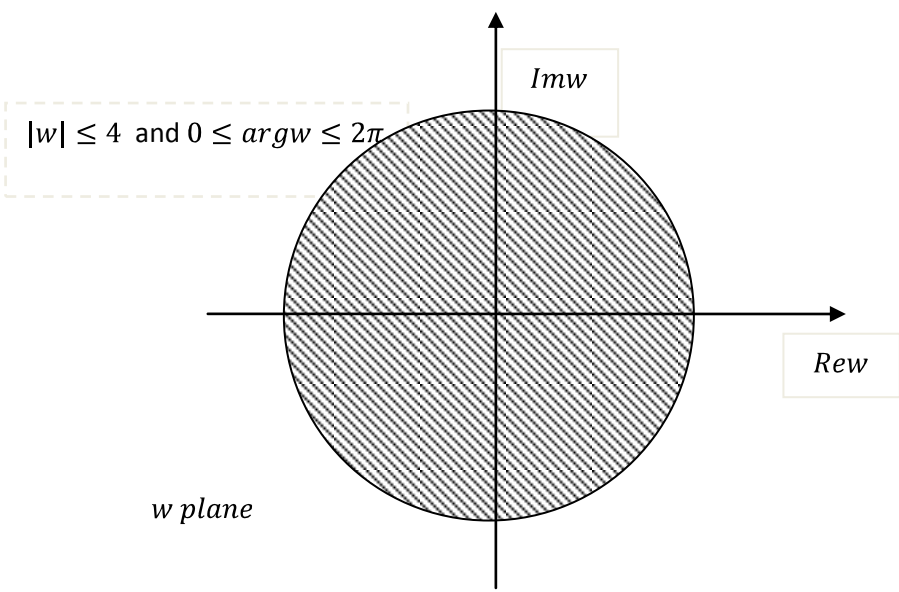
QNo	Answer	Marks	Comments
	$\dot{\theta}^2 = \frac{3g}{a} \left[1 - \sin^2 \left(\frac{\theta}{2} \right) \right] = \frac{3g}{a} \left[\cos^2 \left(\frac{\theta}{2} \right) \right]$ $\dot{\theta} = \sqrt{\frac{3g}{a}} \cos \left(\frac{\theta}{2} \right)$ $\frac{d\theta}{dt} = \sqrt{\frac{3g}{a}} \cos \left(\frac{\theta}{2} \right)$ $\int_0^\theta \frac{d\theta}{\sqrt{\frac{3g}{a}} \cos \left(\frac{\theta}{2} \right)} = \int_0^t dt = [t]_0^t$ $\sqrt{\frac{a}{3g}} \int_0^\theta \sec \left(\frac{\theta}{2} \right) d\theta = t$ $\sqrt{\frac{a}{3g}} \frac{1}{\frac{\theta}{2}} \left[\ln \left(\tan \frac{\theta}{2} + \sec \frac{\theta}{2} \right) \right]_0^\theta = t$ $2 \sqrt{\frac{a}{3g}} \ln \left[\frac{\sin \frac{\theta}{2} + 1}{\cos \frac{\theta}{2}} \right] = t$ $2 \sqrt{\frac{a}{3g}} \ln \left[\frac{\left(\cos \frac{\theta}{4} + \sin \frac{\theta}{4} \right)^2}{\left(\cos^2 \frac{\theta}{4} - \sin^2 \frac{\theta}{4} \right)} \right] = t$ $t = 2 \sqrt{\frac{a}{3g}} \ln \left[\frac{\cos \frac{\theta}{4} + \sin \frac{\theta}{4}}{\cos \frac{\theta}{4} - \sin \frac{\theta}{4}} \right]$ $= 2 \sqrt{\frac{a}{3g}} \ln \left[\frac{1 + \tan \frac{\theta}{4}}{1 - \tan \frac{\theta}{4}} \right]$ $= 2 \sqrt{\frac{a}{3g}} \ln \left[\frac{\tan \frac{\pi}{4} + \tan \frac{\theta}{4}}{1 - \tan \frac{\pi}{4} \tan \frac{\theta}{4}} \right]$ $= 2 \sqrt{\frac{a}{3g}} \ln \left[\tan \left(\frac{\pi + \theta}{4} \right) \right]$		

QNo	Answer	Marks	Comments
5	<p>(a)</p> $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 34e^{3x}$ <p>Take the trial function $y_T = \alpha e^{3x}$</p> $\frac{dy_T}{dx} = 3\alpha e^{3x} \quad \frac{d^2y_T}{dx^2} = 9\alpha e^{3x}$ $9\alpha e^{3x} + 4.3\alpha e^{3x} + 13.\alpha e^{3x} = 34e^{3x}$ $(34\alpha - 34)e^{3x} = 0$ <p>Since $e^{3x} \neq 0$ $\alpha = 1$ $y_T = e^{3x}$</p> <p>The auxiliary equation $\mu^2 + 4\mu + 13 = 0$</p> $\mu = -2 \pm 3i$ <p>The complimentary function $y_c = e^{-2x}(A\cos 3x + B\sin 3x)$ Where A and B are arbitrary constants.</p> <p>The general solution is $y_c = e^{-2x}(A\cos 3x + B\sin 3x) + e^{3x}$</p>	65	
	<p>(b)</p> <p>Suppose $y = \frac{1}{D+\alpha}f(x) \dots\dots\dots [1]$</p> <p>Applying the operator $D + \alpha$ for the both sides</p> $(D + \alpha)y = f(x)$ $\frac{dy}{dx} + \alpha y = f(x)$ <p>The integrating factor $I = e^{\int \alpha dx} = e^{\alpha x}$</p> $e^{\alpha x} \frac{dy}{dx} + \alpha e^{\alpha x} y = e^{\alpha x} f(x)$ $\frac{d(e^{\alpha x} y)}{dx} = e^{\alpha x} f(x)$ $e^{\alpha x} y = \int e^{\alpha x} f(x) dx = e^{-\alpha x} \int e^{\alpha x} f(x) dx$ $y = e^{-\alpha x} \frac{1}{D} e^{\alpha x} f(x) \dots\dots\dots [2]$ <p>By [1] and [2] $\frac{1}{D+\alpha}f(x) = e^{-\alpha x} \frac{1}{D} e^{\alpha x} f(x)$</p>	50	
	$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 5\sin 3x$ $\frac{d}{dx} = D$ $(D^2 + 2D - 2)y = 5\sin 3x$	60	

QNo	Answer	Marks	Comments
	<p>A particular integral</p> $y_p = \frac{1}{(D^2 + 2D - 2)} 5\sin 3x$ $y_p = \frac{1}{(D + 2)(D - 1)} 5\sin 3x$ $y_p = \frac{1}{3} \left(-\frac{1}{(D + 2)} + \frac{1}{(D - 1)} \right) 5\sin 3x$ $= \frac{5}{3} \left(-\frac{1}{(D + 2)} \sin 3x + \frac{1}{(D - 1)} \sin 3x \right)$ $= \frac{5}{3} \left(-e^{2x} \frac{1}{D} e^{-2x} \sin 3x + e^{-x} \frac{1}{D} e^x \sin 3x \right)$ $= \frac{5}{3} \left(-e^{2x} \int e^{2x} \sin 3x \, dx + e^{-x} \frac{1}{D} \int e^{-x} \sin 3x \, dx \right)$ $= \frac{5}{3} \left(e^{-2x} \frac{e^{2x}}{13} [2\sin 3x - 3\cos 3x] - e^{-x} \frac{e^{-x}}{10} [-\sin 3x - 3\cos 3x] \right)$ $= \frac{5}{3} \left(\frac{1}{13} [2\sin 3x - 3\cos 3x] - \frac{1}{10} [-\sin 3x - 3\cos 3x] \right)$ $y_p = \frac{1}{26} [11\sin 3x + 3\cos 3x]$		
	<p>The auxiliary equation $\mu^2 + \mu - 2 = 0$</p> $(\mu + 2)(\mu - 1) = 0$ <p>$\mu = -2$ or $\mu = 1$ Complementary function $y_c = Ae^{-2x} + Be^x$ Where A and B are arbitrary constants.</p> <p>Therefore the general solution is</p> $y = Ae^{-2x} + Be^x + \frac{1}{26} [11\sin 3x + 3\cos 3x]$	2 5	

QNo	Answer	Marks	Comments
6	(a)	70	
	<p>$Re(z + 2) \geq z - 2$</p> <p>Take $z = x + iy$ where $x, y \in R$</p> $Re(x + iy + 2) \geq x + iy - 2 $ $Re(x + 2) \geq x + 2$ $ x + iy - 2 = \sqrt{(x - 2)^2 + y^2}$ <p>When $x + 2 \geq 0$ $x + 2 \geq \sqrt{(x - 2)^2 + y^2}$</p> $(x + 2)^2 \geq (x - 2)^2 + y^2 \quad 8x \geq y^2 \quad y^2 \leq 8x$ <p>$z \leq 4$ is the region inside the circle center origin and the radius 4</p>		

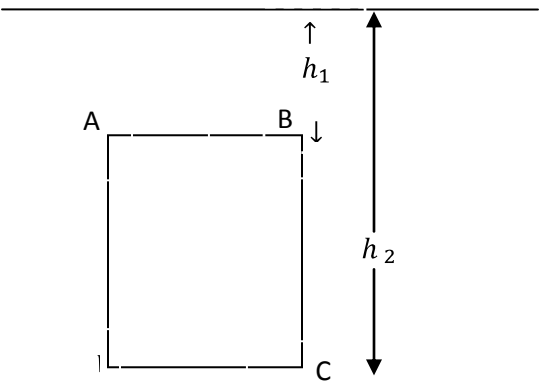
QNo	Answer	Marks	Comments
			
(b)	$1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$ $= \sqrt{2} \left[\cos \left(2k\pi + \frac{\pi}{4} \right) + i \sin \left(2k\pi + \frac{\pi}{4} \right) \right] \text{ where } k \in \mathbb{Z}$ $1 + i = \sqrt{2} e^{(2k\pi + \frac{\pi}{4})i}$ $\text{Log}(1 + i) = \text{Log} \left(\sqrt{2} e^{(2k\pi + \frac{\pi}{4})i} \right) = \ln \sqrt{2} + \left(2k\pi + \frac{\pi}{4} \right) i$ $= \frac{1}{2} \ln 2 + \left(2k\pi + \frac{\pi}{4} \right) i = \frac{1}{2} \ln 2 + \left(2k + \frac{1}{4} \right) \pi i$	45	
(c)	$f(z) = \frac{4z+3}{z-1} \text{ where } z \in \mathbb{C} \setminus \{1\}$ <p>Let $z_1, z_2 \in \mathbb{C}$</p> $f(z_1) = f(z_2) \leftrightarrow \frac{4z_1+3}{z_1-1} = \frac{4z_2+3}{z_2-1}$ $4z_1z_2 - 4z_1 + 3z_2 - 3 = 4z_1z_2 - 4z_2 + 3z_1 - 3$ $z_1 = z_2$ <p>Since z_1 and z_2 are arbitrary</p> $\forall z_1, z_2 \in \mathbb{C} - \{1\} \leftrightarrow f(z_1) = f(z_2) \text{ Therefore } f \text{ is one to one}$ $w = f(z) = \frac{4z+3}{z-1} \quad wz - w = 4z + 3$ $z = \frac{w+3}{w-4} \quad w \neq 4 \quad f^{-1}(w) = \frac{w+3}{w-4} \quad f^{-1}(z) = \frac{z+3}{z-4} \quad z \neq 4$	25	
(d)		60	

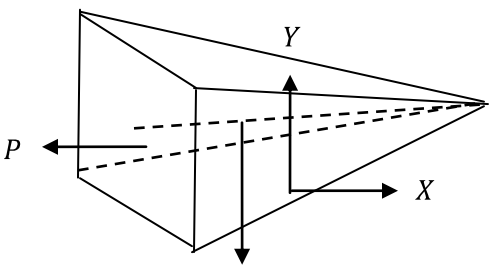
QNo	Answer	Marks	Comments
	<p>Take $g(z) = w = z^2$ $argw = argz^2 = 2argz$ $0 \leq argz \leq \pi$ $0 \leq 2argz \leq 2\pi$ $0 \leq argw \leq 2\pi$ $w = z^2 = z ^2 \leq 16$</p> 		

QNo	Answer	Marks	Comments
7	<p>(a)</p> $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \mu^2 - 14 & \mu + 2 \end{bmatrix}$ <p>Applying $R_2 \rightarrow R_2 - 3R_1$ and $R_3 \rightarrow R_3 - 4R_1$</p> $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & \mu^2 - 2 & \mu - 14 \end{bmatrix}$ <p>Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 - R_2$ and $R_2 \rightarrow \frac{-1}{7}R_2$</p> $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & \mu^2 - 16 & \mu - 4 \end{bmatrix}$ <p>Applying $R_1 \rightarrow R_1 - 2R_2$</p> $\begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & \mu^2 - 16 & \mu - 4 \end{bmatrix}$	125	

QNo	Answer	Marks	Comments
	<p>Case one $\mu^2 - 16 \neq 0$ that is $\mu \neq \pm 4$ Applying $R_3 \rightarrow \frac{1}{\mu^2 - 16} R_3$</p> $\begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{\mu - 4}{\mu^2 - 16} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{1}{\mu + 4} \end{bmatrix}$ <p>Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 + 2R_3$</p> $\begin{bmatrix} 1 & 0 & 0 & \frac{8\mu + 25}{7(\mu + 4)} \\ 0 & 1 & 0 & \frac{10\mu + 54}{7(\mu + 4)} \\ 0 & 0 & 1 & \frac{1}{\mu + 4} \end{bmatrix}$ <p>If $\mu \neq \pm 4$ the system has unique solution $x = \frac{8\mu + 25}{7(\mu + 4)}$, $y = \frac{10\mu + 54}{7(\mu + 4)}$, $z = \frac{1}{\mu + 4}$</p> <p>Case 2 $\mu = -4$</p> $\begin{bmatrix} 1 & 2 & -3 & \frac{4}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 0 & -8 \end{bmatrix}$ then the system is inconsistent no solutions. <p>Case 3 $\mu = 4$</p> $\begin{bmatrix} 1 & 2 & -3 & \frac{4}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ then the system is consistent the solutions and infinitely many solutions. <p>$x = \frac{8}{7} - z$, $y = \frac{10}{7} + 2z$ and z can take any value</p>		
(b)	<p>$\frac{d^2y}{dx^2} + y = 3\sin 2x$ Where $x \geq 0$, $y(0) = 1$ and $\left(\frac{dy}{dx}\right)_{x=0} = 2$.</p> <p>Let $L(y(x)) = Y(s)$</p> $L\left(\frac{d^2y}{dx^2}\right) = s^2Y(s) - sY(0) - \left(\frac{dy}{dx}\right)_{x=0} = s^2Y(s) - s - 2$ $L(\sin 2x) = \frac{2}{s^2 + 2^2}$	75	

QNo	Answer	Marks	Comments
	<p>Taking the Laplace transformation of both side of the equation.</p> $s^2 Y(s) - s - 2 + Y(s) = L(3\sin 2x) = 3L(\sin 2x) = 3 \frac{2}{s^2 + 4}$ $(s^2 + 1)Y(s) = \frac{6}{s^2 + 4} + s + 2$ $Y(s) = \frac{6}{(s^2 + 4)(s^2 + 1)} + \frac{s}{(s^2 + 1)} + \frac{2}{(s^2 + 1)}$ $= \frac{-2}{(s^2 + 4)} + \frac{2}{(s^2 + 1)} + \frac{s}{(s^2 + 1)} + \frac{2}{(s^2 + 1)}$ $Y(s) = \frac{s}{(s^2 + 1)} - \frac{2}{(s^2 + 4)} + 4 \frac{1}{(s^2 + 1)}$ <p>Taking the inverse Laplace transform gives</p> $y(x) = \cos x - \sin 2x + 4\sin x$		

QNo	Answer	Marks	Comments
8.	 <p>The thrust on the strip $PQ = x\rho g dx$</p> <p>The moments of the thrust $FQ = x\rho g dx \cdot x$</p> <p>The total moments on the lamina $= \int_{h_1}^{h_2} ax^2 \rho g dx$</p> <p>The resultant thrust on the lamina $= a(h_2 - h_1) \frac{(h_2 + h_1)}{2} \rho g$</p> <p>The total moments on the lamina $= a(h_2 - h_1) \frac{(h_2 + h_1)}{2} \rho g \bar{x}$</p> $a(h_2 - h_1) \frac{(h_2 + h_1)}{2} \rho g \bar{x} = \int_{h_1}^{h_2} ax^2 \rho g dx$ $(h_2 - h_1) \frac{(h_2 + h_1)}{2} = \left[\frac{x^3}{3} \right]_{h_1}^{h_2}$ $\bar{x}(h_2 - h_1) \frac{(h_2 + h_1)}{2} = \frac{h_2^3 - h_1^3}{3}$	120	

QNo	Answer	Marks	Comments
	$\bar{x} = \frac{2}{3} \left(\frac{h_2^2 + h_1 h_2 + h_1^2}{h_1 + h_2} \right)$  <p>The considering the forces on the liquid cone W</p> <p>Resolving the forces</p> $\omega = 4a^3 \rho g$ $\rightarrow X = 4a^3 \rho g = \omega \quad \uparrow Y = \frac{1}{3} 4a^2 h \rho g = \frac{\omega a}{3h}$ <p>The resultant = $\sqrt{\omega^2 + \left(\frac{\omega a}{3h}\right)^2} = \frac{\omega}{3h} \sqrt{a^2 + 9h^2}$</p> <p>Angle to the horizon $\tan^{-1} \left(\frac{\omega a}{3h} \right)$</p>	80	